On the Nonlinear Continuum Mechanics of the Luminiferous Medium

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In this work, we argue that adding the displacement current in Ampere's law by Maxwell was equivalent to considering the field as an elastic continuum. To corroborate this point, we prove that, when linearized, the governing equations of an incompressible elastic continuum yield Maxwell's equations as corollaries. The divergence of the deviator stress tensor is analogous to the electric field, while the vorticity (the *curl* of velocity field) is interpreted as the magnetic field. The nonlinearity of the material time derivative (the advective part of acceleration) is interpreted as the Lorentz force. Thus we have established that the electrodynamics can be fully explained if one assumes that it is the manifestation of the internal forces of an underlying elastic material which we term the *metacontinuum*.

The possible detection of the absolute continuum is also discussed. First, a new interferometry experiment is proposed in which the first-order Doppler effect can be measured and thus the presence of a medium at rest can be unequivocally established. Second, the famous experiment of Ives and Stilwell is reexamined with a modified Bohr-Rydberg formula for the emitted frequencies from a moving atom, and it is shown that the results of Ives and Stilwell are fully compatible with the presence of an absolute medium.

Keywords: Luminiferous Metacontinuum, Maxwell–Hertz Electrodynamics, Material invariance.

1. Introduction. Is there an Absolute Continuum?

Any dynamic wave phenomenon is associated with either the transverse or longitudinal elastic vibrations of a medium/field; if there is a wave, something material should be 'waving.' This notion led 19th century scientists to introduce the concept of the luminiferous continuum (field, aether, etc.). The first attempt to explain the propagation of light as a field phenomenon was by Cauchy around 1827 (see the account in Whittaker (1989)) who postulated the existence of an elastic continuum, through which light propagates as an elastic shear wave. Unfortunately, Cauchy's model of elastic aether contradicted the natural perception of a particle moving *through* the medium. As a result, his concept did not receive much attention, possibly also because the notion of elastic liquids was not available at that time. Subsequently came the contributions of Faraday and Ampere, which eventually led to the formulation of the electromagnetic model. The crucial advance was achieved, however, when Maxwell (1865) added the term $\frac{\partial \boldsymbol{E}}{\partial t}$ in Ampere's law, which he termed the 'displacement current'. This new term was similar to the time-derivative term in Maxwell's constitutive relation for elastic gases Maxwell (1867) (see also Jordan & Puri (2005) for an insightful discussion on viscoelastic models). Since the electric field vector is an analog of the stress vector in continuum mechanics (see Christov (2006a)), one can say that Maxwell postulated an elastic constitutive relation by adding the displacement current to Ampere's law. Indeed, the new term transformed the system of equations, already established in electrostatics, into a hyperbolic system with a characteristic speed of wave propagation corresponding (mathematically) to the speed of sound in gases. Maxwell identified the characteristic speed of this hyperbolic system with the speed of light, and thus paved the way to understanding the electromagnetic wave phenomena.

The first attempts to detect the absolute medium were based on interferometry experiments (the famous experiment of Michelson and Morley (1887)) but they produced a nil result. Many were quick to interpret the nil result of Michelson and Morley experiment as evidence that the absolute continuum did not exist. Actually, the only thing which the nil result proves is that there is a contraction of the lengths in the direction of motion as suggested by Lorentz who took a hint from what is now called the 'Lorentz transformation' (LT). Thus LT, as a mathematical tool, produced a breakthrough result: it pointed to a possible mechanism for the contraction of moving bodies.

The unfortunate result from the LT and the dismissal of the absolute continuum was the apparent boost to the concept that the inertial frames are somehow equivalent and/or indistinguishable. Poincare generalized Galileo's Relativity Principle and stated that the inertial motion of a frame cannot be discerned by measurements in the same frame (see the account in Pathria (2003)). This is clearly a stretch, because what is obviously true for points and/or systems of points in an empty geometrical space (Galileo's RP) is not necessarily true for the material points of the material space (field) which points interact through the internal stresses.

The Relativity Principle (RP) was aggressively promoted by Einstein despite the fact that Lorentz himself never embraced Poincare-Einstein relativity. The result of the infatuation with relativity principle (and more specifically with the fascinating symmetry of LT), caused the scientists to act as if Maxwell's equations were untouchable. Quite ironically, ten years prior to the advent of RP, the sacred equations had already been changed in the correct direction by Hertz (1900) who proposed to use the convective derivatives in the terms where Maxwell had merely partial time derivative. Instead of making the smallest step to accept that Hertz's equations are valid also *in vacuo*, Einstein and Minkowski took the radical approach to introduce the space-time as the model that somehow ensures the invariance.

Yet, upon closer scrutiny, the postulate of relativity does not automatically appear to be justified by the success of the Lorentz transformation. As elucidated by Brillouin (1970), the relativity theory is fraught with internal logical inconsistencies and riddled by paradoxes. Perhaps, the most obvious and mind-boggling inconsistency is the so-called twin paradox which stems from the prediction of LT that in a moving frame the time slows down. It is quite reasonable to assume that the oscillatory processes (such as atomic clocks) can slow down (change their frequencies) in a frame that is moving relative to the absolute medium where the frequencies are excited. The logical absurdity, however, is that the RP claims that each of the local times is supposed to be slowed relative to each other! Then the question is which time is actually slowed? In other words, the question is of which of the twins will be older upon the return of the one that travelled to the stars. This is a purely kinematic paradox rooted in the paradoxical nature of local time embodied in the LT; however, many people accepted the most unsatisfactory explanation based on the dynamics as provided by Einstein. The explanation was that the time slows in the frame that experiences acceleration. Somehow, this explanation was not challenged by majority of scientists and this, most crippling, paradox gradually became 'old news' and faded away in the collective memory of science, without receiving a consistent explanation.

All this means that while LT and Minkowski space are legitimate mathematical constructs, the relativity principle is still not justified. Actually, the recent measurements of the so-called Local Standard of Rest (LSR) (see, e.g. Anderson (1983), Smoot et al. (1977)) show that there must exist an asymmetry between the moving frames, depending on which one is moving. Even if the motion is inertial, it should be detectable inside of the frame because it *does make a difference* who is moving relative to the underlying absolute continuum. The detection of LSR means that one can safely assume that the motion of an *inertial* frame *can* be detected inside the frame. This directly disproves Poincare's relativity principle. Apparently, coining the new euphemism 'Local Standard of Rest' for the absolute continuum is a way to avoid confrontation with the current orthodoxy in physics.

In the present work we search for the proper model of the absolute continuum, because the 'ethereal,' gas-like substance considered in the Nineteenth century cannot explain the wave phenomena in electrodynamics because light is a shear wave while in gasses, the waves are compressional (longitudinal). We discuss here the issues of material invariance of the absolute continuum as the true covariance in physics. The new concept of invariance replaces the concept of Lorentz covariance which is valid only in non-deforming frames in rectilinear non-accelerating translation. Lorentz covariance is, in fact, a kind of palliative substitute for the real material invariance. The material invariant formulation given here has the same form in any accelerating and even *deforming* material frame.

2. Lorentz Transformation: The Dead-End in the Quest for Invariance

Invariance is the Holy Grail of modern physics. Since Galileo, the question of invariance of any physical description with respect to changing the inertial frame of reference (coordinate system), became the litmus test for the internal consistency of the model. Soon after Maxwell formulated his equations, it was discovered that his model was not invariant with respect to translational motion of the coordinate system. Voigt, and independently Lorentz (see Ernst & Hsu (2001)), spotted the fact that the wave equation can be made invariant, if in the moving frame, the time variable is changed together with the spatial variables. Since the wave equation was believed to describe electromagnetic phenomena in terms of potentials, the non-invariance of the former entailed the non-invariance of Maxwell's electrodynamics.

We present here the argument of Voigt and Lorentz for the scalar potential, ψ, for which a wave equation can be derived *in vacuo*, namely

$$\psi_{tt} = c^2(\psi_{xx} + \psi_{yy} + \psi_{zz}). \tag{2.1}$$

It is enough to consider only the one-dimensional case, when there is no dependence on y and z. The question arises: "Does Eq. (2.1) describe phenomena that are invariant when changing to a moving coordinate frame?" To answer this question, one has to consider the moving frame:

$$x' = x - vt, \quad t' = t \qquad \text{or} \qquad x = x' + vt', \quad t = t',$$

where v is the velocity of the center of the moving coordinate frame. Then the scalar potential in the moving frame is given by $\Psi(x', t') = \psi(x' + vt', t')$, or, alternatively $\psi(x, t) = \Psi(x' - vt', t')$. Upon expressing the partial derivatives of ψ via the partial derivative of Ψ we get

$$\psi_{tt} - c^2\psi_{xx} = \frac{\partial^2\Psi}{\partial t'^2} - 2v\frac{\partial^2\Psi}{\partial t'\partial x'} - (c^2 - v^2)\frac{\partial^2\Psi}{\partial x'^2}, \tag{2.2}$$

which has a different form than Eq.(2.1). This means that in the moving frame, different properties of the electromagnetic waves will be observed. Since nothing of this kind happens in the observations, the conclusion was that something is fundamentally wrong with the whole concept of physics.

One of the casualties of the perceived non-invariance of electrodynamics was the concept of a material medium that pervades the space and serves as the transmitter of the electromagnetic waves. The luminiferous medium was believed to be a very thin ethereal substance (the 'aether'). As it will be shown in the present work, the material continuum is not an ethereal substance although there is no reason to believe that it does not exist. What happened with the dismissal of the aether was that the baby was thrown out with the bath water. If one considers that space is not merely an empty 'geometric' vessel for the physical processes, then one should realize that the time derivative in any kind of motion must be the convective derivative (called in different contexts 'material', 'total', or 'substantial' derivative), not merely the partial time derivative, namely

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}. \tag{2.3}$$

If the convective derivatives are substituted *in lieu* of the local time derivatives, the paradoxical features of the model are instantly removed. It is interesting to note that the concept of convective derivative was around more than a century before Maxwell formulated his equations. Even more, Hertz (1900) proposed to use the convective time derivative in Maxwell's equations when dealing with the electromagnetic field *in* material continua. The Maxwell–Hertz equations are called also 'progressive wave equations' and are clearly invariant because of the material derivative involved.

The Hertz version of the equations of electrodynamics was not appreciated in full because the scientists did not think about space itself as being a material continuum (what we call these days 'physical vacuum'). The question was frequently raised of whether the progressive-wave equations can be construed to hold also *in vacuo*. Contrary to the tenets of the current orthodoxy, the answer is obviously in the affirmative, provided that one accepts the trivial fact that a *physical* vacuum has to be a physical (material) continuum. Unfortunately, Hertz's proposal did not attract much attention, and was discarded on the grounds that the empty space is something different from a mechanical continuum. Thus, the electromagnetic field was assumed not to be a material substance, but rather something else.

As a 'non-material' way out of the quandary of the perceived non-invariance, Voigt and Lorentz proposed to change the time in the moving frame. Their derivations differ from each other by the presence of the Lorentz factor, but both of them (and Larmor and Poincare virtually simultaneously with them), espoused essentially the same idea: the time was no longer an absolute parameter, but could change from frame to frame, being thus influenced by the motion of the frame. What is nowadays called the Lorentz transformation, is the following change of variables

$$x' = \gamma(x - vt), \ t' = t - \frac{v}{c^2}x \quad \text{or} \quad x = \gamma(x' + vt'), \ t = t' + \frac{v}{c^2}x', \tag{2.4}$$

where $\gamma = [1 - v^2/c^2]^{-\frac{1}{2}}$. It is straightforward to demonstrate that this change of variables leaves the linear wave equation invariant, namely

$$\psi_{tt} - c^2\psi_{xx} = \gamma^2\big(1 - v^2/c^2\big)\Big(\frac{\partial^2\Psi}{\partial t'^2} - c^2\frac{\partial^2\Psi}{\partial x'^2}\Big) \equiv \Big(\frac{\partial^2\Psi}{\partial t'^2} - c^2\frac{\partial^2\Psi}{\partial x'^2}\Big). \tag{2.5}$$

The limited success of the Voigt-Lorentz transformation is connected to the fact that it tacitly restores to some extent the convective derivative, i.e. it emulates the material invariance for non-deformable frames in rectilinear motion (see Christov (2006a)). Indeed, in 1D moving frame, the Lorentz transformation gives in the the following

$$\frac{\partial}{\partial t} = \gamma\frac{\partial}{\partial t'} + \gamma v\frac{\partial}{\partial x'}, \tag{2.6}$$

which resembles very much the convective derivative Eq. (2.3), save the contraction factor, γ. Yet, Eq. (2.6) is not equivalent to Eq. (2.3), because the latter is valid for any local velocity (the frame can move non-inertially and even deforming during the motion), while the former is true only for rectilinear motion of the frame. Nobody has been able to generalize the Lorentz transformation for accelerating frames, not to speak about generally deforming frames. This means that as of today the 'covariance' in the term 'Lorentz covariance' is merely wishful thinking and is approximately true only in rectilinearly moving non-deforming frames. For this reason, the present author called the crippled invariance known as 'Lorentz covariance' the *'Poor Man's Material Invariance'* in Christov (2006a). The point of the present work is that the problem with the covariance is by no means solved, and one has to continue the quest for the invariance of electromagnetic phenomena.

One of the most unfortunate developments in twentieth century physics came from the fact that the Lorentz transformation hinted at some superficial symmetry between two inertially moving frames. This prompted Poincare to generalize the idea of Galileo and to state the 'Relativity principle' which postulates that the inertial motion of a frame cannot be detected. While for a system of discrete material points in an empty geometric space, such an assertion does not raise suspicion, in the case of material continuous frames it is quite a stretch. In fact, because of the fact that light *propagates* in the continuum, while the frame is moving with respect to the absolute continuum, one should be able to measure the Doppler effect of electromagnetic waves that are emitted from a source that is at rest with respect to the absolute continuum. This is exactly what was reported in the experiments on the Local Standard of Rest.

Finally, is invariance even possible without demoting the absolute statue of time? The answer is of course 'no', if one looks at the original Maxwell equations. However, the correctly rederived Maxwell-Hertz equations are *material* invariant which hints at the idea that the true covariance lies in the material invariant description (elements of which are called also 'frame indifference' in mechanics of continua). Here we will be guided by the following obvious statement:

"The material world must be material invariant",

and will provide the mathematical technique needed for the theory of absolutivity, which emerges from the fact that there is an absolute continuum. We introduce the concept of the absolute continuum as an elastic body (metacontinuum) and derive the model from the principle of rational continuum mechanics (Chadwick (1999), Truesdell (1965)).

3. The Elastic Metacontinuum

An absolute continuum is more naturally modeled in the frame of material (Lagrange) coordinates $\boldsymbol{X}$. This approach is called the 'referential description' (see, e.g. Chadwick (1999)). The connection of the geometric (Euler) coordinates $\boldsymbol{x}$ in the so-called 'current configuration' is given by

$$\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{X};t). \quad \text{or} \quad \boldsymbol{X} = \boldsymbol{X}(\boldsymbol{x};t). \tag{3.1}$$

The first equation defines, in fact, the trajectory in the geometric space of a given point of the material continuum, as specified by its 'address' $\boldsymbol{X}$ in the material space. Note that $\boldsymbol{x}$ are functions of time while $\boldsymbol{X}$ are independent of time. Hence the explicit dependence on time in Eq. $(3.1)_2$ is needed to 'cancel' the time dependence introduced through the geometric coordinates $\boldsymbol{x}$. It is convenient to denote the gradients in the referential and the current descriptions as

$$\text{grad} \stackrel{\text{def}}{=} \frac{\partial}{\partial \boldsymbol{x}}, \quad \text{Grad} \stackrel{\text{def}}{=} \frac{\partial}{\partial \boldsymbol{X}} \tag{3.2}$$

We follow the standard notations which use capital letter for a differential operation (such as gradient or divergence) in the referential description (material coordinates $\boldsymbol{X}$), and a small letter for the differentiation with respect to the spatial variables.

Without losing the generality, one can assume that at the initial moment of time $t = 0$ the material and geometric coordinates coincide, i.e.

$$\boldsymbol{x}_0 \stackrel{\text{def}}{=} \boldsymbol{x}(\boldsymbol{X};0) = \boldsymbol{X}. \tag{3.3}$$

Then one can define the displacement of a point of the continuum as

$$\boldsymbol{U}(\boldsymbol{X};t) \stackrel{\text{def}}{=} \boldsymbol{x} - \boldsymbol{x}_0 = \boldsymbol{x}(\boldsymbol{X};t) - \boldsymbol{X}. \tag{3.4}$$

Acknowledging the connection between Lagrange and Euler coordinates, Eq. (3.1), one can write the displacement equivalently as a function of the later, namely

$$\boldsymbol{u}(\boldsymbol{x};t) = \boldsymbol{U}[\boldsymbol{X}(\boldsymbol{x};t);t], \tag{3.5}$$

where we use a different case for the letter to stress the point that the functional dependence is now different. Naturally, the numerical values of $\boldsymbol{u}$ are the same as $\boldsymbol{U}$ if the coordinates are related by the transformation formulas, Eq. (3.1).

The velocity of a material point is then the mere time derivative of the displacement when the latter is thought of as a function of the material coordinates and time. The velocity can be expressed as a function of the geometric coordinates, namely

$$\boldsymbol{V}(\boldsymbol{X};t) = \boldsymbol{U}_t(\boldsymbol{X};t), \quad \text{or} \quad \boldsymbol{v}(\boldsymbol{x};t) = \boldsymbol{V}[\boldsymbol{X}(\boldsymbol{x};t);t], \tag{3.6}$$

A most important characteristic is the deformation gradient

$$\mathbb{F} = \text{Grad}\,\boldsymbol{x}\ \left(F_{i\alpha} = \frac{\partial x_i}{\partial X_\alpha}\right) \quad \text{and} \quad \mathbb{F}^{-1} = \text{grad}\,\boldsymbol{X}\ \left(F^{-1}_{\alpha i} = \frac{\partial X_\alpha}{\partial x_i}\right), \tag{3.7}$$

where we reserve the block-capital letters for tensors that are defined simultaneously in the referential and the current configuration.

The Cauchy balance law can be written as (see, e.g., Bland (1960))

$$\rho\frac{D^2\boldsymbol{U}}{Dt^2} \equiv \rho\frac{\partial^2\boldsymbol{U}(\boldsymbol{X};\boldsymbol{t})}{\partial t^2} = \text{div}\sigma = J^{-1}\text{Div}\mathfrak{S} = \frac{\rho}{\mu}\text{Div}\mathfrak{S} \tag{3.8}$$

where D/Dt denotes the material time derivative and Div is the divergence with respect to the Lagrange coordinates. Here, μ is the density in the material reference frame, while ρ is the density in the current description (Euler variables). We have used in Eq. (3.8) the fact that the Jacobian of the deformation gradient gives the equation of continuity, namely

$$J = \det(\mathbb{F}), \qquad \mu/\rho = J. \tag{3.9}$$

The Cauchy stress tensor in the current description is denoted by σ. Respectively, $\mathfrak{S}$ is the Piola-Kirchhoff stress tensor (sometimes called Lagrange stress tensor Bland (1960)) which is related to the Cauchy stress tensor as follows:

$$\mathfrak{S} = J\mathbb{F}^{-1}\sigma. \tag{3.10}$$

We use the ‘Gothic’ fonts for tensors defined solely in the referential description. Now, Eq. (3.8) can be rewritten as

$$\mu\frac{\partial^2\boldsymbol{U}(\boldsymbol{X};\boldsymbol{t})}{\partial t^2} = \text{Div}\mathfrak{S}, \tag{3.11}$$

which is the final form of the equation of motion in the referential description (Lagrange variables).

The main concept developed in our previous works is that the luminiferous continuum is at rest and the particles and charges (see Christov (2005a,2005b,2008b)) are phase patterns in the continuum. As shown in the cited works, the notion of soliton (or quasi-particle) explains the presence of particles and charges (matter) as the phase patterns on the *3D hyper-surface/metacontinuum*. This is the reason to call the luminiferous medium a ‘metacontinuum’, in the sense that it is beyond the matter and is its progenitor.

4. Linear Elasticity and Maxwell's Equations *in vacuo*

In order to keep the size of the paper within the limits we will not consider here the viscous aspect of the metacontinuum. The connection of viscosity to Ohmic resistance is discussed in Christov (2006a, 2007). In this section we focus on the linear elasticity of the metacontinuum in order to elucidate the wave propagation.

The linearized governing equations are valid only for infinitesimal deformations, when the referential and spatial descriptions have the same form, provided that $\boldsymbol{X}$ is thought of as being the same as $\boldsymbol{x}$. The linear constitutive relationship for an elastic body (in the spatial description for definiteness) reads

$$\sigma = (\lambda+\eta)(\mathrm{div}\boldsymbol{u}) + \frac{\eta}{2}(\mathrm{grad}\,\boldsymbol{u} + \mathrm{grad}\,\boldsymbol{u}^T), \tag{4.1}$$

where λ and η are the Lamé coefficients (see, e.g., Segel (1987), Landau & Lifschitz (1986)), and μ is the density of the metacontinuum. The above constitutive law substituted in Eq. (3.8) yields the so-called Navier equations (see, e.g., Segel (1987), p.117) for the displacement vector $\boldsymbol{u}(\boldsymbol{x};t)$

$$\mu\frac{\partial^2 \boldsymbol{u}}{\partial t^2} = (\lambda+\eta)\nabla(\nabla\cdot\boldsymbol{u}) + \eta\nabla^2\boldsymbol{u} = (\lambda+2\eta)\nabla(\nabla\cdot\boldsymbol{u}) - \eta\nabla\times\nabla\times\boldsymbol{u}, \tag{4.2}$$

where we can use the 'nabla' operator ∇ because the equations are written in the current description.

The speeds of propagation of shear ('light') and compressional ('sound') disturbances, are given respectively by

$$c = \left(\frac{\eta}{\mu}\right)^{\frac{1}{2}}, \quad c_s = \left(\frac{2\eta+\lambda}{\mu}\right)^{\frac{1}{2}}, \qquad \delta = \frac{\eta}{2\eta+\lambda} = \frac{c^2}{c_s^2}, \tag{4.3}$$

where the ratio δ is introduced for convenience.

(*a*) *Large Dilational Modulus and Incompressibility of Metacontinuum*

In a compressible elastic medium, both the shear and the dilational/compressional waves should be observable. Since the groundbreaking works of Young and Fresnel, it is well established that electromagnetic waves (e.g., light) are a purely transverse (shear) phenomenon. This observation requires us to reduce the complexity of the model and to find a way to eliminate the term proportional to the dilational modulus λ. Cauchy assumed that $\lambda = 0$ and ended up with the theory of so-called 'volatile aether' (see Whittaker (1989)). Upon a closer examination, we found that such an approach cannot explain Maxwell's equations.

Let us now assume that the other extreme situation is at hand, namely that $\lambda \gg \eta$ which is equivalent to $\delta \ll 1$ or $\delta^{-1} \gg 1$. It is convenient to rewrite Eq. (4.2) in terms of the speeds of sound and light, namely

$$\delta\left(c^{-2}\frac{\partial^2\boldsymbol{u}}{\partial t^2} + \nabla\times\nabla\times\boldsymbol{u}\right) = \nabla(\nabla\cdot\boldsymbol{u}), \tag{4.4}$$

and to expand the density μ, displacements $\boldsymbol{u}$ and velocities $\boldsymbol{v}$ into asymptotic power series with respect to δ, namely

$$\mu = \mu_0 + \delta\mu_1 + \cdots, \quad \boldsymbol{u} = \boldsymbol{u}_0 + \delta\boldsymbol{u}_1 + \cdots, \quad \boldsymbol{v} = \boldsymbol{v}_0 + \delta\boldsymbol{v}_1 + \cdots. \tag{4.5}$$

Introducing (4.5) into (4.4) and combining the terms with like powers we obtain for the first two terms

$$\nabla(\nabla \cdot \boldsymbol{u}_0) = 0\,, \tag{4.6a}$$

$$c^{-2}\frac{\partial^2 \boldsymbol{u}_0}{\partial t^2} + \nabla \times \nabla \times \boldsymbol{u}_0 = \nabla(\nabla \cdot \boldsymbol{v}_1) \stackrel{\text{def}}{=} -\frac{1}{\mu c^2}\nabla\phi\,, \tag{4.6b}$$

where ϕ is introduced for convenience. It plays the same role as the pressure in an incompressible medium, in the sense that it is an implicit function in Eq. (4.6b) that provides the necessary degree of freedom to enforce the satisfaction of the 'incompressibility' condition, Eq. (4.6a). The latter can also be rewritten as

$$\nabla \cdot \boldsymbol{u}_0 = const, \quad \Rightarrow \quad \nabla \cdot \boldsymbol{v}_0 = 0, \tag{4.7}$$

which requires that the velocity field must be solenoidal within the zeroth-order of approximation of the small parameter δ. From now on, the subscript '0' will be omitted form the variables without fear of confusion.

Eq. (4.6b) can be rewritten as

$$\begin{gathered}\mu\frac{\partial \boldsymbol{v}}{\partial t} = -\nabla\phi - \eta\nabla \times \nabla \times \boldsymbol{u} = -\nabla\phi + \boldsymbol{t}, \\ \boldsymbol{t} \stackrel{\text{def}}{=} \nabla \cdot \sigma = -\eta\nabla \times \boldsymbol{u}, \quad \text{for} \quad \nabla \cdot \boldsymbol{u} = const,\end{gathered} \tag{4.8}$$

where $\boldsymbol{v}$ is the velocity vector and $\boldsymbol{t}$ is the tangential part of the stress vector in the metacontinuum. The normal part of the stress vector is given by the gradient of the potential ϕ, the latter being proportional to the pressure/tension in the medium.

(*b*) *Maxwell's Equations as Corollaries*

The system, Eq. (4.8), lends itself to a far-reaching analogy if one terms the negative stress vector the 'electric force,' and defines the 'magnetic field,' $\boldsymbol{H}$, as the vorticity in the metacontinuum, namely

$$\boldsymbol{E} \stackrel{\text{def}}{=} -\boldsymbol{t} = \eta\nabla \times (\nabla \times \boldsymbol{u}), \qquad \boldsymbol{H} \stackrel{\text{def}}{=} \nabla\times\boldsymbol{v}, \quad \boldsymbol{B} = \mu\boldsymbol{H}. \tag{4.9}$$

where the magnetic induction is defined the usual way for the electrodynamics *in vacuo*.

Now taking the *curl* of Eq. (4.8) and acknowledging the notations (4.9), we get Faraday's law

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}. \tag{4.10}$$

On the other hand, taking the time derivative of Eq. $(4.9)_1$ we get

$$\frac{1}{\eta}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times (\nabla \times \boldsymbol{v}) \equiv \nabla \times \boldsymbol{H}, \tag{4.11}$$

or in terms of magnetic induction

$$\frac{\mu}{\eta}\frac{\partial \boldsymbol{E}}{\partial t} = \nabla \times \boldsymbol{B} \quad \Rightarrow \quad \frac{\partial \boldsymbol{E}}{\partial t} = c^2\nabla \times \boldsymbol{B}. \tag{4.12}$$

which is exactly the second of Maxwell's equation. The latter is properly termed the 'Maxwell-Ampere' equation, because Maxwell added the displacement current $\boldsymbol{E}_t$ in Ampere's equation.

(c) Effect of Compressibility (Dark Energy?)

There are no conceptual difficulties to extend the model to include a slight compressibility of the metacontinuum. Such a generalization raises the question about the speed of the compressional waves ('sound') of the metacontinuum. This speed is expected to be much greater (δ^{-1} times greater) than the speed of the shear waves, c (in the visible spectrum the shear waves are called light). However, the investigation of the quantitative effects of a slight compressibility goes beyond the scope of the present paper. In order to avoid ambiguous terminology we will not use the term *sound* for the compression waves in the metacontinuum. Rather we will call them the *pneuma*.

One obvious implication of the existence of waves of a different kind than the shear (electromagnetic) waves is that, in fact, there is more energy in the physical vacuum than detected from electromagnetic radiation. The *pneuma* waves are orthogonal to the shear waves and are not detectable by electromagnetic based devices. They perfectly fit the bill of what is currently called 'dark energy' (see Huterer & Turner (1999)). Over the last couple of decades, observations of supernovae have shown that the expansion of the Universe is accelerating (Perlmutter (1999), Riess (1998)). One way to quantitatively model this acceleration is to 'play' with the cosmological constant (Peebles & Ratra (2003)). Yet, a physical (mechanical) cause for the cosmological constant to change is still needed. Recently, a consensus has emerged around the conjecture that the Universe is expanding faster than expected because of the presence of (what some have been termed 'negative') pressure that acts through space and pushes the matter apart.

It is important to investigate the interaction between the shear and longitudinal vibrations. For instance, the transfer of energy under certain conditions from the longitudinal (undetectable) mode to the shear (detectable) mode may turn out to be the cause of unexplained sources of electromagnetic radiation (e.g., gamma-rays bursts). In the linear approximation the *pneuma* waves and the electromagnetic waves do not couple, so the next stage of the present theory will be to include the effects of finite elasticity in the constitutive relations.

(d) Summary of the Incompressible Metacontinuum

In concluding this section, we mention that Eq. (4.8) governs the propagation of transverse (shear) waves and accomplishes the goal of Cauchy who attempted to explain light waves as shear waves of a material medium. We also mention that in the case of elastic liquids, η corresponds to ζ/τ, where ζ is the shear coefficient of viscosity and τ is the relaxation time. At this point, it is still not unequivocally clear if one is faced with a viscoelastic liquid (Maxwell liquid) or with a viscoelastic body (Voigt–Kelvin body) whose elastic limit is given by Eq. (4.2). The connection between the description of the present section and Christov (2007) can be established by observing that for the propagation of the shear waves, an elastic liquid and an elastic body behave essentially in the same manner. The possible difference could be observed if one investigates a metacontinuum that can actually flow. This, however, goes beyond the scope of the present work.

5. Localized Torsional Dislocations in the Metacontinuum

In order to understand the interplay between the absolutivity of the metacontinuum and the relativity of the rectilinear motion, as discussed by Einstein (1961) in Ch. 7, we consider a $2D$ case with only two nonzero displacement components, $u_x(x, y)$ and $u_y(x, y)$ and no motion of the *material* frame, i.e., no predominant velocity component. In this case, we can introduce a 'displacement function' in the same fashion as the stream function is introduced for incompressible flows. Then, for the two components of displacement we have $u_x = \partial_y\psi$, $u_y = -\partial_x\psi$, and from the linearized governing equations (4.2) the following wave equation can be derived

$$\partial_{tt}\psi - c^2(\partial_{xx} + \partial_{yy})\psi = 0. \tag{5.1}$$

It is interesting to note that the last equation has a stationary solution with polar symmetry: $\psi = \ln(r)$, where $r = \sqrt{x^2 + y^2}$, which is a localized solution for the component B_z of the magnetic induction. For the displacement components, one has the following expressions $u_x = y/r^2, u_y = -x/r^2$. This solution represents the well known potential vortex (in the left panel of Fig. 1) which has a nontrivial circulation (topological charge). This suggests that the localized torsional deformations can be interpreted as the charges. In what follows, we term this kind of a localized wave pattern the 'twiston.' Note that a localized vortex-like solution can also be obtained for the velocity (see Christov (2007)). Then $\boldsymbol{v}$ has the same functional dependence as the above outlined solution for $\boldsymbol{u}$. To distinguish between the *twiston* of velocity (the well known potential vortex), we refer to the *twiston* of the displacement as the 'gnarl'.

Now, we examine the situation when a localized solution of the above type *propagates* with phase velocity $\boldsymbol{w} = (0, w)$. There are no essential difficulties to consider a more general moving frame $\boldsymbol{w} = (w_1, w_2)$, but we resort to the simple translation along the y-axis for the sake of simplicity. Note that even for a large phase speed, $|\boldsymbol{w}| \lessapprox c$, the actual magnitudes of the displacements of the material points of the metacontinuum are still very small, and thus one can use the linearized equation, Eq. (5.1). Then, in the moving frame $\hat{x} = x$, $\hat{y} = y - wt$, one gets from Eq. (5.1) that $\psi_{\hat{x}\hat{x}} + (1 - w^2)\psi_{\hat{y}\hat{y}} = 0$, which possesses a solution of the type

$$\psi = \ln z, \quad z = \sqrt{\hat{x}^2 + \hat{y}^2(1 - w^2/c^2)^{-1}}. \tag{5.2}$$

We observe that the analytic form of the solution is the same, but for a different radial-like coordinate, z. The lines of constant z are ellipses, i.e. one is faced with a 'gnarl' whose displacement lines are ellipses. This means that the moving phase pattern, Eq. (5.2), undergoes contraction in the direction of motion proportional to the Lorentz factor. This situation is depicted in the right panel of Fig. 1. Since, the potential of interaction of two localized waves (see Christov & Christov (2008)) for a similar derivation in the case of *sine*-Gordon equation) depends on the asymptotic behavior of their 'tails,' the fact that the charges are shortened in the direction of motions will lead to shortening of the distances between them in the same direction. This means that an assemblage of charges (i.e., a body) will be shortened in the direction of motion by the Lorentz factor.

Thus, the relativity of rectilinear motion of phase patterns and the *Lorentz contraction* are manifestations of the *absolutivity of space*. The patterns *propagate*

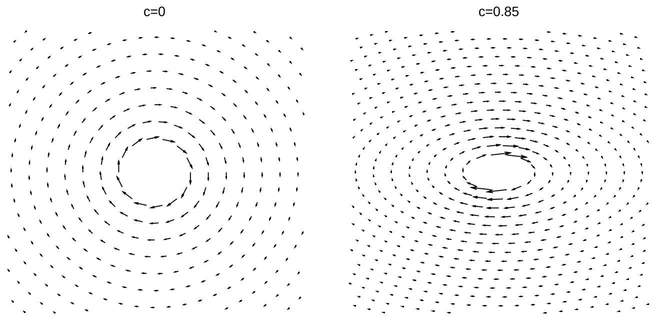

Figure 1. A localized gradient phase pattern (twiston) in two dimensions. Left panel: pattern at rest. Right panel: Pattern propagating with phase velocity (w_1, w_2).

over, rather than *move through*, the *metacontinuum* and do not create an 'aether wind,' i.e. the rectilinear translation of phase patterns does not contradict the principle of material invariance since the latter holds for the points of the material metacontinuum. The above derivations have a limited quantitative importance due to their 2D nature, but their qualitative significance is very important because they show the interplay between the notions of absolute continuum and relative rectilinear motion.

An important caveat is due her connected with the fact that the solution (5.2) is singular at the center of the coordinate system. Singularity is not a thing which is easy to reconcile with the common sense. A similar singularity was encountered in the famous Schwarzschild's solution for the 'black holes'. Einstein considered the Schwarzschild singularity as an artificial constructs. Similarly, in the case of the *twiston*, the Einstein's objection is equally valid. Moreover, that while in general theory of relativity, the investigators are not bound by a mechanical construct and can freely speculate about singularities, here we must provide, at least, an outline of the way out of this situation.

As shown in Christov (2005a), the improper behavior can be mitigated if higher-order derivatives are present in the models. In elasticity, this is called 'gradient elasticity' (see the original work of Mindlin (1964), and the illuminating presentation in Sharma & Ganti (2005), from the point of view of a general Lagrangian formalism for the higher-gradient elasticity). The issue of higher derivatives involved in viscous liquids is thoroughly elucidated by Jordan & Puri (2002) and Quintanilla & Straughan (2005) in the context of dipolar and Green & Nahdi's (1970) liquids.

Now, Eq.(3.7) of Sharma & Ganti (2005), Eq.(2.1) of Quintanilla & Straughan (2005), and Eq.(2.10) of Jordan & Puri (2002), all of them give the following generic equation for the stationary *twiston* with the higher-gradients are acknowledged:

$$\Delta \boldsymbol{u} - \chi \Delta\Delta \boldsymbol{u} = 0, \tag{5.3}$$

where χ is the coefficient of the higher-gradient elasticity or viscosity. Once again upon introducing the 'displacement function' (or alternatively the 'stream function'), $u_x = \frac{\partial \psi}{\partial y}$ and $u_y = -\frac{\partial \psi}{\partial x}$ in 2D, we can reduce eq.(5.3) for the case of radial symmetry to the following

$$\frac{1}{r}\frac{d}{dr} r \frac{d}{dr}\left[\chi \frac{1}{r}\frac{d}{dr} r \frac{d}{dr}\psi - \psi\right] = 0, \tag{5.4}$$

which is shown in Christov (2005a) to possess the following solution

$$\psi = K_0(r/\sqrt{\chi}) + \ln(r/\sqrt{\chi}). \tag{5.5}$$

The last equation does not have a singularity in the origin since $K_0(r) \propto -\ln r$ for small r. This means that the higher-gradient effect have to be considered in a more practical model of the metacontinuum. Since the higher-gradient terms play the role of dispersion, the question arises if there are some observations that suggest that dispersion could be present in the metacontinuum. The best candidate for a dispersion related effect is the redshift. Christov (2001) showed that the higher gradient can actually lead to stretching the internal length of an elastic solitary wave. Alternatively, Christov (2008b) found that the dissipation can make a narrow wave packet redshift (change its central wave number) without changing much its apodization function. This means that in a metacontinuum with higher-gradient elasticity or viscosity, a cosmological redshift will necessarily be present.

The detailed description of the higher-gradient generalizations goes beyond the scope of the present work.

6. Euler Variables and Material Invariance

As already discussed in Section 2, the laws of physics (including the laws of continuum physics) must have the same form in any reference frame (coordinate system). Unlike what is called 'Lorentz Covariance', the laws in referential description are *material* invariant, i.e. they are truly covariant. However, the experimental measurements are always connected with a current frame in the geometric space. This means that an observational frame comprised by quasi-particles (phase patterns) cannot detect the material variables, but rather can merely measure their counterparts in the current (geometric) frame. This is a typical situation in mechanics of continuous media where the reference configuration is often not related to any measurable frame. For this reason we need to reformulate the model from Section 3 in the current description making use of Euler variables. This is the objective of the present section.

In terms of the velocity vector, the Cauchy balance, Eq. (3.8), can be rewritten as follows:

$$\rho\frac{D\boldsymbol{v}}{Dt} \stackrel{\text{def}}{=} \mu\frac{\partial \boldsymbol{v}}{\partial t} + \mu\boldsymbol{v}\cdot\nabla\boldsymbol{v} = \text{div}\sigma = -\nabla\phi - \boldsymbol{E}, \tag{6.1}$$

where D/Dt stands for the total derivative (material derivative in the current configuration. Remember that in the referential configuration, it is just the partial time derivative. Note also that for an incompressible metacontinuum the density is the same constant in both the referential and spatial descriptions and we denoted it by μ. Also, we use the above defined stress vector related to the deviatoric part of the stress tensor, which we have called the 'electric field', $\boldsymbol{E}$. The Cauchy balance, Eq. (6.1) can be rewritten in the so-called 'Lamb form'

$$\mu\frac{\partial \boldsymbol{v}}{\partial t} - \mu\boldsymbol{v}\times\text{curl}\,\boldsymbol{v} = -\text{grad}\big(\phi + \frac{\boldsymbol{v}^2}{2}\big) - \boldsymbol{E}, \tag{6.2}$$

Now, taking the *curl* of Eq. (6.2), and using our definitions Eq. (4.9), we get:

$$\text{curl}\,[\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}] = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{6.3}$$

which is the Faraday law with the Lorentz force, $\boldsymbol{v} \times \nabla \boldsymbol{B}$, already accounted for. This is a very important result, because it tells us that the Lorentz force is not an additional, empirically observed force that has to be grafted on the Maxwell model, but is connected to the material time derivative, and hence it is a manifestation of the material invariance of the model. Under the incompressibility condition, Eq. (6.3) can be recast to

$$\frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{B} = -\mathrm{curl}\, \boldsymbol{E}, \tag{6.4}$$

which we can call Faraday–Lorentz–Hertz law.

The next step in building the new concept is to understand the second of Maxwell's equations and its relationship with the constitutive relation. The physically relevant quantity here is not the stress tensor itself, but the total force acting on the surface S_t surrounding an arbitrary volume V_t in the current configuration, namely

$$\boldsymbol{t} = \iint_{S_t} \boldsymbol{\sigma n}\, \mathrm{d}a = \iint_{S_r} \mathfrak{S} \boldsymbol{N}\, \mathrm{d}A = \int_{V_r} \mathrm{Div}\, \mathfrak{S} \mathrm{d}V, \tag{6.5}$$

where $\mathfrak{S}$ is the Piola-Kirkhoff stress-tensor, defined in Eq. (3.10). Here $\boldsymbol{n}$ stands for the outer normal vector and da – for the elementary area in the spatial description. Respectively, $\boldsymbol{N}$ and $dA$are the normal vector and the elementary area in the referential description.

The constitutive law for an elastic body gives the relationship between the stress and strain tensors. This relationship can be formulated either in the referential or in the spatial description. The main difference between them stems from the finite elasticity, when it is assumed that the deformations are not infinitesimal. Since acknowledging the finite elasticity will obscure the main idea of the present work, so we resort to a linear relationship between the stress and strain tensors; and what is more important, we keep only the linear terms in the expression of the strain tensor via the deformation gradient $\mathbb{F}$. Thus, for the linearized elasticity of an incompressible continuum we get

$$\mathrm{Div}\, \mathfrak{S} = (\lambda + 2\eta) \mathrm{Grad}\, \mathrm{Div} \boldsymbol{U} - \eta\, \mathrm{Curl}\, \mathrm{Curl}\, \boldsymbol{U} = -\eta\, \mathrm{Curl}\, \mathrm{Curl}\, \boldsymbol{U}. \tag{6.6}$$

Recall that for an incompressible metacontinuum Grad Div $\boldsymbol{U} = 0$.

Note that assuming linear elasticity (very small deformations) does not automatically entail small velocities in the spatial description, so it is meaningful to derive the material-invariant time derivative in the spatial description.

The idea when using the current (spatial) representation is to have in the model only variables that can be measured in the current frame, such as velocities and accelerations. As already mentioned, the deformations depend also on the referential frame which is very often unknown. For this reason we take the material time derivative of Eq. (6.5) to get

$$\frac{D\boldsymbol{t}}{Dt} = -\eta \frac{D}{Dt} \int_{V_r} \mathrm{Curl}\, \mathrm{Curl}\, \boldsymbol{U} \mathrm{d}V = -\eta \int_{V_r} \mathrm{Curl}\, \mathrm{Curl}\, \frac{\partial}{\partial t} \boldsymbol{U} \mathrm{d}V. \tag{6.7}$$

We can interchange the material derivative and the integration in the r.h.s. of the above equality because the integral is taken in the referential frame where the partial time derivative is equivalent to the material derivative. Being reminded of the definition of magnetic induction, Eq. (4.9), we recast the above equality as follows

$$\frac{D\boldsymbol{t}}{Dt} = -\eta \int_{V_r} \text{Curl Curl} \frac{\partial}{\partial t}\boldsymbol{U}\mathrm{d}V = -\eta \int_{V_r} \text{Curl}\, \dot{\boldsymbol{U}}\mathrm{d}V = -\frac{\eta}{\mu}\int_{V_r} \boldsymbol{B}\mathrm{d}V. \tag{6.8}$$

In turn, the material time derivative of the stress vector integrated over the moving volume can be transformed as follows (a similar derivation for $\phi\boldsymbol{n}$ can be found in Chadwick (1999)):

$$\frac{D\boldsymbol{t}}{Dt} = \frac{d}{dt}\iint_{S_t} \boldsymbol{\sigma n}\,\mathrm{d}a = \iint_{S_r} \frac{\partial}{\partial t}\left[\boldsymbol{\Sigma}\{J(\mathbb{F}^{-1})^T\boldsymbol{N}\}\mathrm{d}A\right] = \iint_{S_r} \frac{\partial}{\partial t}\left[\boldsymbol{\Sigma}J(\mathbb{F}^{-1})^T\right]\boldsymbol{N}\mathrm{d}A$$

$$= \iint_{S_r} \frac{\partial}{\partial t}\left(J\mathbb{F}^{-1}\boldsymbol{\Sigma}\right)\boldsymbol{N}\mathrm{d}A = \iint_{S_r} \left\{\dot{J}\mathbb{F}^{-1}\boldsymbol{\Sigma} + J(\mathbb{F}^{-1})^{\cdot}\boldsymbol{\Sigma} + J\mathbb{F}^{-1}\dot{\boldsymbol{\Sigma}}\right\}\boldsymbol{N}\mathrm{d}A, \tag{6.9}$$

where $\boldsymbol{\Sigma}$ is the Cauchy stress tensor σ as a function of the material derivatives:

$$\boldsymbol{\Sigma}(\boldsymbol{X};t) \equiv \boldsymbol{\sigma}[\boldsymbol{x}(\boldsymbol{X};t);t]. \tag{6.10}$$

We observe that

$$\dot{\mathbb{F}} = \text{Grad}\,\dot{\boldsymbol{x}} = (\text{grad}\,\dot{\boldsymbol{x}})\mathbb{F} = \mathbb{L}\mathbb{F}, \qquad \mathbb{L} \stackrel{\text{def}}{=} \text{grad}\,\dot{\boldsymbol{x}} = \text{grad}\,\boldsymbol{v}, \tag{6.11a}$$

$$\mathbb{F}(\mathbb{F}^{-1})^{\cdot} = -\dot{\mathbb{F}}\mathbb{F}^{-1} = -\mathbb{L}, \quad \Rightarrow \quad (\mathbb{F}^{-1})^{\cdot} = -\mathbb{F}^{-1}\mathbb{L} = -\mathbb{L}^T\mathbb{F}^{-1}. \tag{6.11b}$$

$$\dot{J} = J\text{tr}\mathbb{L} = J\text{div}\,\boldsymbol{v}. \tag{6.11c}$$

Upon introducing all the above equalities in the r.h.s. of Eq. (6.9) we get

$$\frac{d\boldsymbol{t}}{dt} = \iint_{S_r} \left\{J\text{tr}(\mathbb{L})\mathbb{F}^{-1}\boldsymbol{\Sigma} - J\mathbb{L}^T\mathbb{F}^{-1}\boldsymbol{\Sigma} + J\mathbb{F}^{-1}\dot{\boldsymbol{\Sigma}}\right\}\boldsymbol{N}\mathrm{d}A$$

$$= \iint_{S_r} \left\{\text{tr}(\mathbb{L})\boldsymbol{\Sigma} - \mathbb{L}^T\boldsymbol{\Sigma} + \dot{\boldsymbol{\Sigma}}\right\} J(\mathbb{F}^{-1})^T\boldsymbol{N}\mathrm{d}A \equiv \iint_{S_t} [(\text{div}\boldsymbol{v})\boldsymbol{\sigma} - \boldsymbol{\sigma}\text{grad}\boldsymbol{v} + \dot{\boldsymbol{\sigma}}]\boldsymbol{n}\mathrm{d}a$$

$$= \int_{V_t} \text{div}[(\text{div}\boldsymbol{v})\boldsymbol{\sigma} - \boldsymbol{\sigma}\text{grad}\,\boldsymbol{v} + \dot{\boldsymbol{\sigma}}]\mathrm{d}v. \tag{6.12}$$

where the last integral is obtained after the divergence theorem is used in the current frame. Now

$$\begin{aligned}
&\text{div}[(\text{div}\boldsymbol{v})\boldsymbol{\sigma} - \boldsymbol{\sigma}\text{grad}\boldsymbol{v} + \dot{\boldsymbol{\sigma}}] \\
&\quad = (\text{div}\boldsymbol{v})\text{div}\boldsymbol{\sigma} + \boldsymbol{\sigma}\text{grad}(\text{div}\boldsymbol{v}) - \boldsymbol{\sigma}\text{grad}(\text{div}\boldsymbol{v}) - (\text{div}\boldsymbol{\sigma})\cdot\text{grad}\boldsymbol{v} + (\text{div}\boldsymbol{\sigma})^{\cdot} \\
&= -[(\text{div}\boldsymbol{v})\boldsymbol{E} - \boldsymbol{E}\cdot\text{grad}\boldsymbol{v} + \dot{\boldsymbol{E}})] = -[\boldsymbol{E}_t + \boldsymbol{v}\cdot\text{grad}\boldsymbol{E} - \boldsymbol{E}\cdot\text{grad}\boldsymbol{v} + (\text{div}\boldsymbol{v})\boldsymbol{E})].
\end{aligned}$$

The last expression is nothing else but the upper-convected (Oldroyd) derivative of a vector density $\boldsymbol{E} \stackrel{\text{def}}{=} -\text{div}\boldsymbol{\sigma}$.

Introducing the above results in Eq. (6.8) we obtain

$$\int_{R_t} [\frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{v}\cdot\text{grad}\boldsymbol{E} - \boldsymbol{E}\cdot\text{grad}\boldsymbol{v} + (\text{div}\boldsymbol{v})\boldsymbol{E} - c^2\boldsymbol{B}]\mathrm{d}v = 0 \tag{6.13}$$

Since the volume R_t is arbitrary, the above equation is satisfied only if the integrand is equal to zero. One can be reminded here that for an incompressible metacontinuum one has div$\boldsymbol{v} = 0$. The following differential equation can be derived from Eq. (6.13):

$$\frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{v} \cdot \text{grad}\boldsymbol{E} - \boldsymbol{E} \cdot \text{grad}\boldsymbol{v} = c^2 \boldsymbol{B}. \tag{6.14}$$

It should be noted here that the last equation is the generalized Maxwell-Ampere equation derived in Christov (2006a), where the Euler description of the material derivative of a vector density (upper-convected Oldroyd derivative) has been used.

In summary, the material invariant form of electrodynamics is governed by Eqs. (6.4), (6.14). In Christov (2006a), this system has been shown to be Galelean invariant. This should come as no surprise because the Galilean invariance is a very limiting case of the material covariance.

Note that in this work we left aside the problems connected with the current *in vacuo*. We mention that the current (and the resistance) are connected with the viscous part of the rheology and it was elucidated in Christov (2006a, 2007) in the framework of the model of Maxwell (elastic) liquids. The connection to the Kelvin-Voigt viscoelastic body will be discussed elsewhere.

7. Detecting the Absolute Continuum

(*a*) *Interferometry Methods: Michelson and Morley Experiment*

Maxwell (1875) proposed to use split-beam interferometry to detect the absolute continuum observing that the effect will be of second order with respect to the relative speed of the frame (the Earth) through the aether. Michelson undertook the challenging task and after various attempts and refinements, he and Morley reported nil effect from the measurements Michelson & Morley (1886, 1987). After some prolonged and controversial discussions, the conclusion was reached that Earth's motion relative to the aether was not detected because an absolute medium simply does not exist.

Actually, in the interpretation of the Michelson and Morley Experiment (MME) a logical fallacy, commonly known as *ignoratio elenchi*, occurred. It consists of using an argument that is supposed to prove one proposition, but succeeds only in proving a different one. In the case of the Michelson-Morley experiment, the fallacy manifests itself in the fact that from the nil effect, the only rigorous conclusions that can be drawn are in the reverse order of their generality:
(i) the luminiferous continuum is not detectable by means of interferometry experiment that involves a single light source, beam splitting, and reflections from mirrors;
(ii) the luminiferous continuum exists, but is not detectable in principle;
(iii) there exists no absolute luminiferous continuum.

In the beginning of the Twentieth century, having apparently ignored the first two cases (and some corollaries form them), the investigators embraced the third choice, abandoning the concept of material carrier of the electromagnetic field altogether, and thus ripping the whole of the fabric of physics. The irony of this unfortunate fallacy is much sadder, because the first choice was implemented in the contraction hypothesis of FitzGerald and Lorentz according to which the expected second-order effect is *canceled* by the contraction of the material space in the direction of motion. The contraction of the scales means that that it is impossible to detect the luminiferous continuum by an experiment based on splitting of the beam. The Lorentz contraction has been splendidly confirmed and verified in numerous high-quality experimental works. Nobody paid attention that of the three options, (i) satisfies Occam's razor which means that this situation can be explained without invoking (ii) and/or (iii). The iconoclastic spirit of the first decade of the twentieth century was so strong that the requirement for logical consistency of the arguments conceded the ground to light-cavalry reasoning motivated by the desire to reject the old concepts.

This being said, the onus is still on all of us to accept or reject (or at least attempt verifying) (ii) and (iii). In this work, we argue that it is possible to verify (ii). In order to do this, we have to devise a fist-order experiment which can show that the interferometry results are compatible with the presence of an unmovable medium between elements of the interferometry experiment. The only way to pass judgment on the detectability of the absolute continuum is to derive a formula for the interference that is based on the assumption that the space between the different parts of the equipment is filled with a continuous medium in which the propagation speed of linear waves is a given constant.

(*b*) *A First-Order Interferometry Experiment*

As already mentioned, it is a generally accepted view now that MME cannot be used to detect the absolute medium because the expected second-order effect is exactly canceled by the Lorentz contraction. As shown in Section 5, the Lorentz contraction is actually one of the strongest arguments in favor of the existence of an absolute medium rest. Then any experiment that uses split-beam interferometry is theoretically capable of detecting only a second-order effect (Maxwell (1875)). Thus, the following question arises: is it at all possible to detect the metacontinuum by means of an interferometry experiment whose parts are moving together with the Earth? The answer (as originally suggested in Christov (1996) and elaborated in Christov (2006b)) is in the affirmative, provided that one can use two *independent* sources of light of virtually identical frequencies and avoid reflections. This means that one has to aim the beams against each other, as shown in Fig. 2. Note that

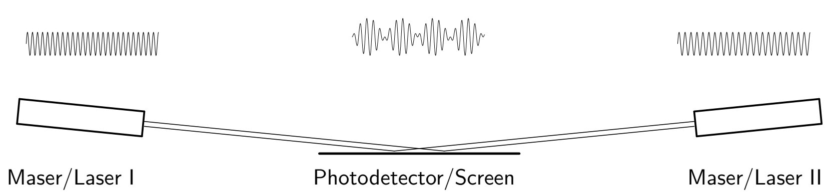

Figure 2. Experimental set-up involving two lasers/masers

using two lasers does not make our experiment similar to the set-up in Jaseda et al. (1963) because the latter involves split-beam and mirrors seeking the second-order effect; and thus, like the Michelson–Morley experiment, it cannot be used to detect the absolute continuum.

Now assume that two waves of identical frequencies are excited at two *different* points the latter moving together in the same direction with the same velocity relative to the resting medium. The interference between the right-going wave from the left source and the left-going wave from the right source is given by

$$\begin{aligned}\exp\left[i\omega(t-x/c)/(1-u/c)\right]+\exp\left[i\omega(t+x/c)/(1+u/c)\right]\\ =\left[\cos(\omega_1 t-k_1 x)+\cos(\omega_2 t+k_2 x)+i[\sin(\omega_1 t-k_1 x)+\sin(\omega_2 t+k_2 x)\right]\\ =2\cos(\tilde{\omega}t+\hat{k}x)\exp\left[i(\hat{\omega}t+\tilde{k}x)\right],\end{aligned} \quad (7.1)$$

where
$$\tilde{\omega}=\frac{\omega_1+\omega_2}{2}=\omega\left(1-\frac{u^2}{c^2}\right), \quad \hat{\omega}=\frac{\omega_2-\omega_1}{2}=-\frac{u}{c}\tilde{\omega},$$

are the carrier and beat frequencies, and $\tilde{k}=\tilde{\omega}/c$, $\hat{k}=\hat{\omega}/c$. The wave excited at a certain point, say $x=0$, is

$$2\cos\left(\tilde{\omega}t\right)\exp\left(i\hat{\omega}t\right). \quad (7.2)$$

One way to find the beat frequency is to use a photodetector at a point in the region of interference of the two waves. Note that the carrier frequency of visible light is very high and cannot be detected by a photodetector. The problem is that even the beat time frequency for the first-order effect, Eq. (7.2), is too high for the resolution of the available photodetectors.

The high beat frequency could be the reason why it was not detected in the experiments of Jaseda et al. (1963,1964) as an certain 'unwanted' modulation of the signal. In fact, they were seeking the beat frequency connected with the second-order effect and found practically no beat, which is exactly what is to be expected for a dynamical medium at rest. It is accepted in the literature that the speed of the LSR to which our solar system belongs, is of order $v \approx 300$ km/s relative to the center of the local cluster of galaxies (Smoot et al. (1977),). The speed of the LSR is an estimate of the speed with respect to the resting metacontinuum, i.e.. $\varepsilon = v/c \propto 10^{-3}$. For red-light lasers with frequency around 500 THz, the beat frequency is expected to be around 500 GHz, which is well beyond the sensitivity of the available photodetectors.

The other way to conduct the experiment is to measure the beat wave number $\hat{k}$ by taking a snapshot of the wave at a certain moment of time. Then the spatial distribution of the wave profile is

$$2\cos(\hat{k}x)\exp i(\tilde{k}x),$$

which will produce an interference pattern in the resting continuum that can be observed on a screen (as shown in Fig. 2). Note that in this case the screen is 'parallel' or 'tangent' to the vibrating part of the dynamical continuum, and what is observed are the dark and light strips corresponding to the different values of the amplitude of the beat wave. For instance, since red light has a wavelength approximately in the range of 600m^{-9}, then the beat wave length is expected to be $\varepsilon^{-1} \approx 1000$ times longer. This gives 0.6mm for the fringes which is technically feasible to observe on a screen. The effect will be best observed if the two laser beams have identical polarization but the case of different polarization can easily be accommodated in the formulas. Also, the requirements for the frequency stabilization of the sources of light stem from from the magnitude of the effect and is safe to ask for stabilization of $\varepsilon \times 10^{-3} \propto 10^{-6}$, which is very easy to attain with the modern day commercial lasers.

(*c*) *Emission of Light by Moving Atoms*

Another important advance of our understanding of the absolute continuum stems from the understanding that in the classical theories of interferometry, the light was considered merely as a known abstract ('mathematical') wave whose frequency is a known number. Such an approach would be good if one were able to measure instantaneously the frequency with the help of a detector that is at rest with respect to the absolute continuum. Clearly, this is an idealization that cannot happen in a moving frame (Earth). This is because all probes used to measure frequencies are moving with the frame too. In this section, we will examine the physical process of emitting of light by moving atoms.

A photon is emitted/absorbed when an electron jumps from one orbit to another in an atom. If the atom is at rest with respect to the absolute continuum, then the frequency is defined by the distance, d, between the orbits for the particular jump. If so, the spatial wave number of an emitted photon is

$$\nu = R\Big(\frac{1}{n_f^2} - \frac{1}{n_i^2}\Big), \quad R = \frac{2\pi^2 e^4 m}{ch^3}\mathrm{cm}^{-1} \approx 109737\mathrm{cm}^{-1}, \tag{7.3}$$

where ν is the spatial wave number, and R is the Rydberg number (see e.g. Joos (1986)). The inverse of the Rydberg number defines the above mentioned length $d \sim R^{-1}$, which gives the measure of the distance between the orbits.

If the atom (a complex phase pattern) is in relative motion with respect to the metacontinuum, we have shown in Section 5 that its dimensions are contracted in the direction of motion. This means that the distance between two orbits under consideration is shortened by the Lorentz factor, i.e., the distance becomes $d^* = d\sqrt{1 - v^2/c^2}$, where v is the speed of the relative motion of the atom with respect to the metacontinuum.

Note, however, that changing the distance between the orbits (Lorentz contraction) is not the only effect for atoms moving with respect to the absolute medium. The conjecture that the atom is a phase pattern that propagates *over* the metacontinuum has an even more important implication. Specifically, the actual length over which the photon is emitted is not d, and not even $d^* = \overline{AB} = \overline{A'B'}$, but actually the length $\overline{AB'}$ (see Fig. 3).

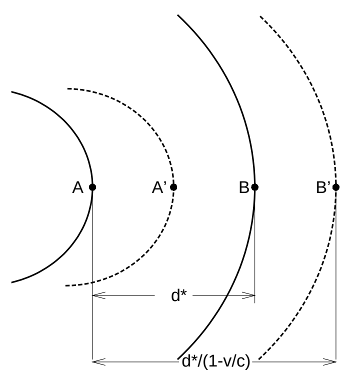

Figure 3. Bohr-Rydberg emission model for a moving atom.

Now, suppose that T is the time needed for a photon to traverse the distance between points A and B', i.e., $cT = \overline{AB'}$. Then $\overline{BB'} = vT$, where, v is the speed of the phase pattern (atom).

$$cT = \overline{AB'} = \overline{AB} + \overline{BB'} = d^* + vT,$$

which gives

$$T_m\big(1 - \frac{v}{c}\big) = \frac{d}{c}\sqrt{1 - \frac{v^2}{c^2}} \quad \Rightarrow \quad T_m = \frac{d}{c}\frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}, \quad \omega_m = \frac{2\pi}{T_m} = \frac{2c\pi}{d}\frac{\sqrt{1-v/c}}{\sqrt{1+v/c}}, \tag{7.4}$$

where and $\omega = 2c\pi/d$ is the frequency of the respective spectral line that would have been emitted in a frame that is in complete rest with respect to the absolute continuum. It is important to point out here that Eq. (7.4) has the same form as what is called relativistic Doppler effect (see, e.g. Gill (1965)). Similarly to the case with Lorentz contraction, this formula is demonstration of absolutivity of space, and need not be explained through relativistic addition of velocities.

(*d*) *Ives-Stilwell Experiment*

In Ives & Stilwell (1938,1941) experiment, atoms are emitted in a cathode tube and the light from the atoms in motion interferes with the light from the atoms that are at rest with respect to the experimental frame (cathode tube). According to the above derivations, the frequency ω_2 of the waves emitted by the speeding atoms and the frequency of light emitted by the atoms at rest are

$$\omega_e = \omega\frac{\sqrt{(1+u/v)}}{\sqrt{(1-u/c)}}, \quad \omega_r = \omega\frac{\sqrt{(1+v/c)}}{\sqrt{(1-v/c)}}. \tag{7.5}$$

where $u = v + q$ is the total speed of the emitting atoms, v is the unknown speed of Earth with respect to the metacontinuum, and q is the relative speed of the atoms while moving within the cathode tube. Within the second order of approximation of u/c (or v/c and q/c, for that matter) one gets

$$\begin{aligned}\frac{\omega_e}{\omega_r} &= \frac{\sqrt{(1+u/v)(1-v/c)}}{\sqrt{(1+v/c)(1-u/c)}} = \left[1 - \tfrac{1}{2}\frac{(v+q)^2}{c^2}\right]\left[1 + \frac{v+q}{c} + \frac{(v+q)^2}{c^2}\right] \\ &\times \left[1 - \frac{v}{c}\right]\left[1 + \tfrac{1}{2}\left(\frac{v}{c}\right)^2 + \dots\right] = \left[1 + \frac{v+q}{c} + \tfrac{1}{2}\frac{(v+q)^2}{c^2}\right]\left[1 - \frac{v}{c} + \tfrac{1}{2}\left(\frac{v}{c}\right)^2\right] \\ &= 1 + \frac{v+q-v}{c} + \tfrac{1}{2}\frac{(v+q)^2 + v^2 - 2v(v+q)}{c^2} = 1 + \frac{q}{c} + \tfrac{1}{2}\frac{q^2}{c^2}.\end{aligned} \tag{7.6}$$

The most important result here is that the velocity of the Earth, v, does not show up in the final formula! After subtracting the zeroth-order and the first-order terms (as Ives and Stilwell did), one finds that the frequency is increased by the factor $\frac{1}{2}q^2/c^2$, which is usually hailed as confirmation of time dilation and considered as another confirmation of the demise of the aether. Our derivations show that this is another conclusion that ignores the fact that an alternative explanation exists based on the assumption that an absolute luminiferous medium exists.

The absolute motion is very elusive, indeed; no surprise that it hasn't been detected until now. The conclusion of this section is that a properly understood absolute continuum can perfectly explain the results of the Ives-Stilwell experiment. At least, it should be given the proper consideration along with the hypothesis of time dilation.

8. Conclusion

In this paper we have argued the case for existence of an absolute material continuum in which the electromagnetic vibrations propagate. The logical fallacies of the arguments that led to the so-called relativity principle are examined and the conclusion is reached that space is actually a material elastic continuum which we call the *metacontinuum*.

The linearized governing equations of the metacontinuum are shown to yield the well known Maxwell's equations as corollaries. Through judicious distinction between the referential and current descriptions, the principle of material invariance is established and shown to be a true covariance principle, unlike the Lorentz covariance, which is valid only for non-deforming frames in rectilinear relative motion. Then the new formulation of the electrodynamics is shown to incorporate the Lorentz force as an integral part of the model, rather than as an additional empirical variable. An immediate corollary of the material invariance is the Galilean invariance of the model.

The charges (and particles in general) are considered as localized phase patterns that propagate *over* the metacontinuum in the same fashion as a wave propagates over the sea surface. The motion of a material point of metacontinuum does not have to be in the direction of the propagation of the wave; and hence no 'aether wind' can be expected. The most important property of the propagating patterns (well known from the theory of solitons) is that the patterns contract in the direction of motion. We show here that the contraction is proportional to the Lorentz factor, i.e. the Lorentz contraction is explained as a manifestation of the absolute continuum.

The problem of detecting the absolute continuum is also addressed. First, a new interferometry experiment is proposed in which the first-order Doppler effect can be measured directly, and thus the Earth's relative motion with respect to the absolute continuum can be detected. Second, the Bohr-Rydberg formula for the frequency of the emitted photon is reformulated to include the effects connected with the motion of the emitting atom relative to the absolute continuum. The new formula is applied to the famous experiment of Ives and Stilwell and shown that it can be, indeed, explained from the point of view of the absolute continuum without invoking the hypothesis of time dilation.

Thus a consistent model of space as an elastic material continuum has been formulated and shown to explain the main known experimental observations. In a sense, the proposed approach can be called the 'Special Theory of Absolutivity.'

References

Bland, D. R. 1960, *Theory of Linear Viscoelasticity.* Pergamon, New York.

Brillouin, L. 1970 *Relativity Reexamined.* Academic Press, New York.

Chadwick, P. 1999 *Continuum Mechanics. Concise Theory and Problems.* Dover, Mineola, New York.

Christov, C.I. 1996 Discrete out of continuous: Dynamics of phase patterns in continua. *In: Continuum Models and Discrete Systems – Proceedings of CMDS8*, K. Markov, editor, pages 370–394, Singapore, World Scientific.

Christov, C. I. 2001 On the Analogy Between the Maxwell Electromagnetic Field and the Elastic Continuum, *Annual of University of Sofia*, **95**, 109–121.

Christov, C. I. 2005a Dynamics of patterns on elastic hypersurfaces. Part I. Shear Waves in the Middle Surface. In *"ISIS International Symposium on Interdisciplinary Science", Natchitoches, October 6-8, 2004*, pages 46–52, Washington D.C., APS Conference Proceedings 755.

Christov, C. I. 2005b Dynamics of patterns on elastic hypersurfaces. Part II. Wave mechanics of flexural quasi-particles. In *"ISIS International Symposium on Interdisciplinary Science", Natchitoches, October 6-8, 2004*, pages 53–60, Washington D.C., APS Conference Proceedings 755.

Christov, C.I. 2006a On the material invariant formulation of Maxwell's displacement current. *Found. Physics*, **36**, 1701–1717.

Christov, C.I. 2006b Much ado about nil: Reflection from moving mirrors and the interferometry experiments. *Progress in Physics*, **3**, 55–59.

Christov, C. I. 2007 Maxwell-Lorentz electrodynamics as a manifestation of the dynamics of a viscoelastic metacontinuum. *Math. Comput. Simul.*, **74**, 93–103.

Christov, C. I. 2008a On the Evolution of Localized Wave Packets governed by a Dissipative Wave Equation, *Wave Motion*, **45**, 154–161.

Christov, C. I. 2008b The concept of a quasi-particle and the non-probabilistic interpretation of wave mechanics. *Math. Comp. Simul.*, submitted.

Christov, I. & Christov, C. I. (2008) Physical dynamics of quasi-particles in nonlinear wave equations. *Phys. Lett. A*, **372**, 841–848.

Einstein, A. 1961 *Relativity. The Special and the General Theory.* Three Rivers Press, New York.

Ernst, A. & Hsu, J. P. 2001 First proposal of the universal speed of light by Voigt in 1887. *Chinese J. Phys.*, **39**, 211–230.

Gill, T. P. 1965 *The Doppler Effect*, Logos Press, London.

Green, A. E. & Naghdi, P. M. 1970 A note on dipolar inertia. *Q. Appl. Math.*, **28**, 458–460.

Hertz, H. 1900 *Electric Waves.* MacMillan, London.

Huterer, D. & Turner, M. S. 1999 Prospects for probing the dark energy via supernova distance measurements. *Phys. Rev. D*, **66**, Art. 081301.

Ives, H. E. & Stilwell, G. R. 1938 An experimental study of the rate of moving atomic clock. *J. Optical Soc. Am.*, **28**, 215–226.

Ives, H. E. & Stilwell, G. R. 1941 An experimental study of the rate of moving atomic clock. II. *J. Optical Soc. Am.*, **31**, 369–374.

Jaseda, T. S, A. Javan, A. & Townes, C. H. 1963 Frequency stability of he-Ne masers and measurements of length. *Phys. Rev. Lett.*, **10**, 165–167.

Jaseda, T. S., Javan, A., Murray, J, & Townes, H. C. 1964 Test of special relativity or of the isotropy of space by use of infrared masers. *Phys. Rev. A*, **113**, 1221–1225.

Joos, G. 1986 *Theoretical Physics.* Dover, New York, Second Ed.

Jordan, P. M & Puri, A. 2005 Revisiting Stokes first problem for Maxwell fluids. *Quart. Jl. Mech. Appl. Math.*, **58**, 213–227.

Jordan, P. M. & Puri, P. 2002 Exact solutions for the unsteady plane Couette flow of a dipolar fluid, *Proc. R. Soc. Lond. A*, **458**, 1245–1272.

Landau, L. D. & Lifschitz, E. M. 1986 *Theory of Elasticity*. Butterworth-Heinemann, 3rd edition.

Maxwell, J. C. 1865 A dynamical theory of the electromagnetic field. *Phil. Trans. Roy. Soc. London*, **155**, 469–512.

Maxwell, J. C. 1867 On the dynamical theory of gases. *Phil. Trans. Roy. Soc. London*, **157**, 49–88.

Maxwell, J. C. 1875 Ether. In *Enciclopedae Britanica. Ninth Edition*, volume VIII, pages 568–572.

Michelson A. A. & Morley, E. W. 1886 Influence of motion of the medium on the velocity of light. *Am. J. Sci.*, **31**, 337.

Michelson, A. A. & Morley, E. W. 1887 On the relative motion of the earth and the luminiferous ether. *Am. J. Sci.*, **34**, 333–345.

Pathria, R. K. 2003 *The Theory of Relativity*. Courier Dover.

Peebles, P. J. E. & Ratra B. 2003 The cosmological constant and dark energy. *Rev. Mod. Phys.*, **75**, 559–606.

Perlmutter, S. 1999 Measurements of omega and lambda from 42 high redshift supernovae. *Astrophysical J.*, **517**, 565–586.

Quintanilla, R. & Straughan, B. (2005) Bounds for some non-standard problems in porous flow and viscous Green–Naghdi fluids, *Proc. R. Soc. A*, **461**, 3159–3168.

Riess, A. G. 1998 Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astronomical J.*, **116**, 1009–38.

Segel, L. A. 1987 *Mathematics Applied to Continuum Mechanics*. Dover, New York.

Sharma P. & Ganti S. 2005 Gauge-field-theory solution of the elastic state of a screw dislocation in a dispersive (non-local) crystalline solid, *Proc. R. Soc. A*, **461**, 1081–1095.

Smoot, G. F,, & Gorenstein, M. V. & Miller, R. A. 1977 Detection of anisotropy in the cosmic blackbody radiation. *Phys. Rev. Lett.*, **39**, 898–901.

Truesdell, C. (1965) *Continuum Mechanics. Vol.I-IV*. Gordon and Breach, New York.

Whittaker, E. T, 1989 *A History of the Theories of Aether & Electricity vol. 1.* Dover, New York, 1989