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General characteristics of radiations emitted by systems moving with super-light velocities with some applications to plasma physics

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The mechanism of radiation of light by a system moving with a super-light velocity is a very simple one and common to the radiation at corresponding conditions of all kinds of waves - electromagnetic as well as sound waves, waves on the surface of water, etc.

Consider a system which in principle is able to emit the radiation in question - e.g. an electrically charged particle in the case of light, a projectile or an airplane in the case of sound, etc. As long as the velocity of this system as a whole is smaller than the velocity of propagation of waves in the surrounding medium, the radiation can be produced only by some oscillatory motion of the system or of some of its parts - e.g. by the oscillation of an electron in an atom or by the revolutions of the propellers of a plane. The frequency of the radiation emitted is evidently determined by the frequency of the oscillations in question. To be more exact, for the radiation to be possible the motion has not necessarily to be a periodic one, but it has to be non-uniform* (i.e. its velocity should not be constant in time).

But when a velocity of the system becomes greater than that of the waves in question, quite a new mechanism of radiation is introduced, by means of which even systems possessing a constant velocity radiate. Let c'(w) denote the velocity of propagation in the surrounding medium of waves, possessing the frequency ω. Then as a rule the radiation of a system moving in the medium with a constant velocity ν, embraces all the frequencies which satisfy the fundamental condition

$$n > c'(w) \tag{1}$$

* About an exception *to* this rule - the so-called transition radiation - see **V. L. Ginzburg** and I. Frank[1] (1945).

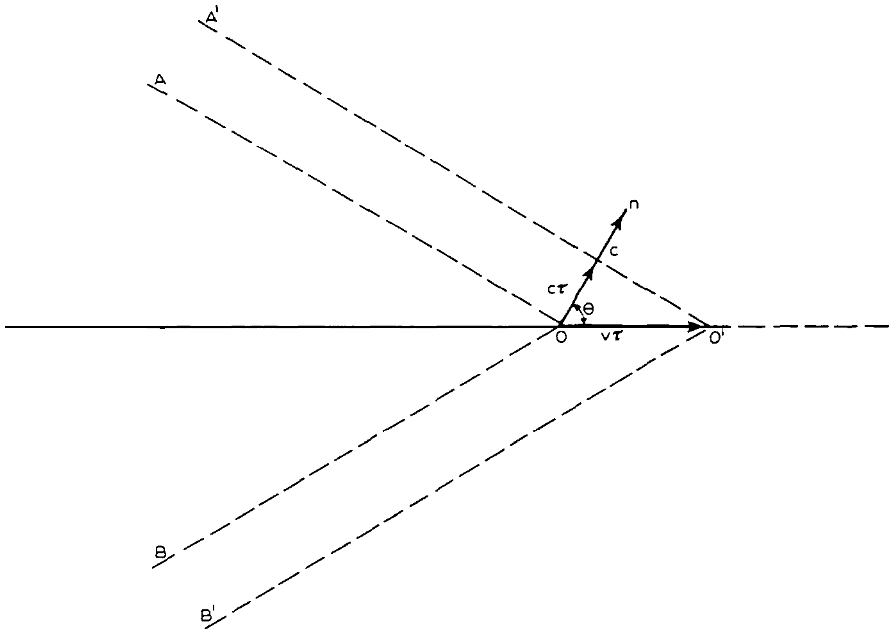

Fig. 1.

This radiation is characteristically a very directional one - waves of a given frequency ω are emitted only under a definite angle Θ to the direction of motion of the system, this angle being determined by the relation

$$\cos \Theta = \frac{c'(\omega)}{v} \qquad (2)$$

To prove these fundamental relations one has only to take account of the fact that at all velocities, whether small or large, the field of a uniformly moving system must be stationary with respect to this system. If the system radiates, it means that in its field at least one free wave is present (a free wave of a frequency ω is by definition propagated in the medium with the characteristic phase velocity c'(w) to any distance, however far from the source of the wave). Let O and O' (Fig. 1) be the positions of the uniformly moving system at two consecutive moments $t = 0$ and $t = \tau$. The phase of the wave radiated by the system must be stationary with respect to the system. It means, that if AO is that front of the wave* which at the moment

* The fronts of the wave are conical, due to the cylindrical symmetry; AOB is the projection on the plane of drawing of such a cone.

$t = 0$ passes through the system at O, then this front, being propagated in the medium with the velocity $c'(\omega)$, will permanently keep up with the system, and in particular will at the moment $t = \tau$ occupy such a position AO', as to pass through O'. Now the direction $\vec{n}$ of propagation of a free wave is perpendicular to its front, therefore the triangle OCO' is a rectangular one and we easily obtain from it the fundamental relation (2).

Since the value of a cosine cannot exceed unity, Eq. (1) follows directly from (2).

All these general properties of the radiation in question were for a very long time well known in aerodynamics. The air waves emitted at supersonic velocities are called Mach waves. The emission of these waves sets in when the velocity of a projectile or of a plane begins to exceed the velocity of sound in the air. Emitting waves means losing energy and these losses are so large that they constitute the main source of resistance to the flight of a supersonic plane.

That is why in order to cross the sound barrier, i.e. to achieve supersonic velocities in aviation, it was necessary to increase very substantially the power of the engines of a plane.

We perceive the Mach waves radiated by a projectile as its familiar hissing or roaring. That is why, having understood the quite similar mechanism of the Vavilov-Čerenkov radiation of light by fast electrons, we have nicknamed it « the singing electrons ».

I should perhaps explain that we in the USSR use the name « Vavilov-Čerenkov radiation » instead of just « Čerenkov radiation » in order to emphasize the decisive role of the late Prof. S. Vavilov in the discovery of this radiation.

You see that the mechanism of this radiation is extremely simple. The phenomenon could have been easily predicted on the basis of classical electrodynamics many decades before its actual discovery. Why then was this discovery so much delayed? I think that we have here an instructive example of a situation not uncommon in science, the progress of which is often hampered by an uncritical application of inherently sound physical principles to phenomena, lying outside of the range of validity of these principles.

For many decades all young physicists were taught that light (and electromagnetic waves in general) can be produced only by *non-uniform* motions of electric charges. When proving this theorem one has - whether explicitly or implicitly - to make use of the fact, that super-light velocities are forbidden by the theory of relativity (according to this theory no material body can ever even attain the velocity of light). Still, for a very long time the theorem was considered to have an unrestricted validity.

So much so, that I. Frank and I, even after having worked out a mathematically correct theory of Vavilov-Čerenkov radiation, tried in some at present incomprehensible way to reconcile it with the maxim about the indispensibility of acceleration of charges. And only on the very next day after our first talk on our theory in the Colloquium of our Institute we perceived the simple truth: the limiting velocity for material bodies is the velocity of light *in vacuo* (denoted by c) whereas a charge, moving in a *medium* with a constant velocity v, will radiate under the condition $v > c'(\omega)$, the quantity $c'(\omega)$ depending on the properties of the medium. If $c'(\omega) < c$, then this condition may very well be realized without violating the theory of relativity $(c' < v < c)$.

When we first discussed our theory with Professor A. Joffe he pointed out to us that A. Sommerfeld[2] as long ago as 1904 has published a paper, dealing with the field of an electron possessing a constant velocity greater than that of light, and has calculated the resistance to such a motion, due to the radiation, emitted by the electron. But Sommerfeld considered only the motion of an electron *in vacuo*. A year later the theory of relativity came into existence, the motion considered by Sommerfeld was proved to be impossible, Sommerfeld's paper was completely forgotten and for the first time in many years was referred to in our papers of the year 1937.

Let us return now to general characteristics of radiation emitted at superlight velocities. In addition to those already indicated a new and very peculiar one emerged in papers of I. Frank[4] in 1943 and of V. L. Ginzburg and I. Frank[5] of 1947.

Suppose that a system A, moving with a constant velocity $\vec{v}$, radiates an amount of energy ε in a direction characterized by a unit vector $\vec{n}$. The balance of energy gives the relation

$$\varepsilon + \Delta T + \Delta U = 0 \tag{3}$$

where ΔT and ΔU denote respectively the increase, caused by the radiation, of the kinetic energy T of the translational motion of the system A and of the energy U of its internal degrees of freedom. On the other hand, if the radiated energy ε is propagated in the medium with the velocity c'

in a definite direction $\vec{n}$, it necessarily possesses a momentum* ε/c', directed along $\vec{n}$. Therefore the conservation of momentum leads to the vector equation

$$(\varepsilon/c')\vec{n} + \Delta\vec{p} = 0 \tag{4}$$

where $\vec{p}$ is the momentum of the system A. If the increase $\Delta\vec{p}$ of $\vec{p}$ is small in relation to $\vec{p}$, then, according to a general rule,

$$\vec{v}\cdot\Delta\vec{p} = \Delta T \tag{5}$$

Combining these simple and general relations one gets

$$\Delta U = -\varepsilon\left(1 - \frac{v\cos\Theta}{c'}\right) \tag{6}$$

where Θ is the angle between $\vec{v}$ and $\vec{n}$.

If the system A possesses no internal degrees of freedom (e.g. a point charge), then $\Delta U = 0$ and Eq. (6) reduces to the already discussed Eq. (2). Thus we have obtained this fundamental equation once again, but by a new way of reasoning. On the other hand, if the system possesses internal (say, oscillatory) degrees of freedom, and if its velocity is small $(v \ll c')$, then, usual, the internal energy U of the system decreases by an amount equal to the amount ε of the energy radiated.

But at super-light velocities $(v > c')$ the value of the bracket in (6) may become negative, so that radiation of energy by the system may be companied by a *positive* increase $(\Delta U > 0)$ of its internal energy U. example, an atom, being originally in the stable state, radiates light an the same time becomes excited! In such a case the energy both of then tion and of the excitation is evidently borrowed from the kinetic energy i.e. the self-excitation of a system is accompanied by a corresponding slowing down of the motion of this system as a whole.

*For the case of electromagnetic radiation it was shown first by quantum-theoretical reasoning (Ginzburg, 1940) and then by means of classical electrodynamics (Marx and Györgyi[6], 1955) that ε/c' (c' being the phase velocity) is in fact equal to the total

The relation (6) emerged in discussion of optical problems but it is of a quite general nature and it may turn out to be useful to apply it in aerodynamics (just as Mach's aerodynamical relations (1) and (2) turned out to be useful in optics).

Certainly, a correct calculation of a supersonic motion will automatically take in account everything, including the possible self-excitation of some particular modes of vibrations of a supersonic plane. However, such calculations are necessarily extremely complicated, so that the relation (6) may prove to be useful in giving an insight in the general mechanism of some of the phenomena which become possible at supersonic velocities. On the other hand, Eq. (6) takes in account only the radiative damping of oscillations, whereas in the case of mechanical vibrations of a plane this kind of damping is under ordinary conditions quite negligible in comparison with the damping caused by the internal friction in the vibrating materials. In short, we must consider it an open question whether the phenomena indicated may be of any importance in the complicated problem of a supersonic flight.

Let us now consider as an example some applications of the general theory to a special field, namely to plasma physics.

In a preparatory way we begin with some remarks on the mechanism of energy losses, experienced by fast charged particles travelling through matter. Vavilov-Cerenkov's radiation accounts only for a part - and usually a very small part - of these losses, which are largely due to the ionization and excitation of the medium traversed by the particles. However the mathematical treatment, used by Frank and myself to calculate the radiation losses, proved to be useful for the general problem also and was extended in 1940 by Fermi[7] so as to cover the total energy loss of a charged particle, with the exception of the losses caused by head-on collisions of the particle with atoms of the medium. The losses of the later kind must be calculated separately. The main difference between Fermi's work and ours is that we assumed the medium traversed by the particle to be transparent, whereas Fermi took in account not only the polarization of the medium by the electrical field of the particle, as we did, but also the absorption of electromagnetic waves in it. Fermi has shown, that the screening of the field of the particle, which is caused by the polarization of the medium, and which was not taken in account in previous work on this subject, very considerably reduces the energy losses of very fast particles.

We will not review here the very extensive work on the subject, in which

Fermi's theory was further elaborated and extended. But to obtain some insight into the underlying mechanism we will consider in some detail the processes taking place in a plasma (e.g. a highly ionized gas), which for our purposes may be considered as the simplest of all media. I have myself not done any work on this subject, so that I will report on the work of others, mentioning by name the authors only of relatively new papers, without explicit references to classical works such as e.g. by N. Bohr.

Energy losses of a charged particle traversing plasma can be divided in two parts. Imagine a cylinder of a radius equal to the Debye's radius $D = (\varkappa T/4\pi Ne^2)^{\frac{1}{2}}$, the axis of the cylinder coinciding with the path of the particle. The interaction of the particle considered with plasma particles lying inside the cylinder must be treated microscopically; resulting energy losses will be referred to as those due to close collisions. But the interaction of the particle considered with the plasma lying *outside* the cylinder can be treated macroscopically; resulting energy losses will be designated as coherent ones. Under ordinary conditions losses of both kinds are of about equal importance, but in a very hot and rarefied plasma, so important in thermonuclear research, the cross section for the direct Coulomb interaction of charged particles decreases and the coherent losses eventually become preponderant.

Since the index of refraction n of a plasma is for all frequencies less than 1, so that the velocity of light $c' = c/n$ in plasma is greater than its velocity c in vacuo, it may appear that the Vavilov-Čerenkov effect should be absent in plasma. But that is not the case. Firstly, only the velocity $c'(\omega)$ of transverse electromagnetic waves in a plasma exceeds *c at all frequencies,* but not so the velocities of plasma waves proper. Those are longitudinal waves, in which oppositely charged plasma particles oscillate in opposite directions, the restoring force being provided by the resulting electric field. Secondly, in a magnetic plasma, i.e. in a plasma exposed to an external magnetic field, both kinds of waves become interconnected, so that no sharp distinction can be drawn between the transverse and the longitudinal waves. As a result the index of refraction of light varies with the directions of its propagation and polarization, and in a certain range of these directions becomes greater than 1, so that the Vavilov-Cerenkov effect becomes possible.

Let us first consider coherent energy losses of a charged particle moving in a plasma in the absence of external magnetic fields. Almost all these coherent losses are due to the excitation of longitudinal plasma waves by a mechanism equivalent to the mechanism of Vavilov-Čerenkov radiation of light. To be more exact the phase velocity of plasma waves is equal to

$$c' = \sqrt{3v_T{}^2 + \frac{\omega_0{}^2}{k^2}}$$

where $k = 2\pi/\lambda$ is the wave vector,

$$\omega_0 = \left(\frac{4\pi\, Ne^2}{m}\right)^{\frac{1}{2}}$$

the so-called plasma frequency and v_T is the mean thermal velocity of plasma electrons. As long as the velocity v of the particle considered is less an $\sqrt{3}v_T$, the necessary condition $v > c'$ for the emission of plasma waves cannot be satisfied; and therefore practically all energy losses experienced by the particle are due to close collisions. But when v exceeds $\sqrt{3}v_T$ the condition $v > c'$ is satisfied for a certain range of wavelengths $\lambda = 2\pi/k$ and the coherent losses are switched in*.

Allow me now to make a digression and to turn your attention from plasma to solid metals. At high enough frequencies the valence electrons in a metal can be considered as free and thus as forming together with the atom cores a kind of plasma. The plasma frequency ω_0 is proportional to the square root of the density of plasma electrons. Since this density is in a metal far greater than in an ordinary plasma, the frequency of plasma waves in metals is rather high, of the order of $h\omega \sim 10$ eV.

In analogy to the case of an ordinary plasma we have to expect that a fast electron traversing a metal foil will experience, besides other kinds of energy losses, also losses due to the excitation of plasma waves by the mechanism just described. Now that is in fact the case. It is well known that fast electrons traversing a thin metal foil often experience in it large discrete energy losses of the order of 10 eV. I refer you to a comprehensive article by D. Pines[8] (1956), where it is shown that an elementary theory of the plasma excitation in a metal by a fast charged particle, very similar to the theory outlined above for the case of an ordinary plasma, fits the experimental facts relating to discrete energy losses in metals so well, that, in words of the author: "What puzzles exist have to do with why the

*The fact that long plasma waves are very strongly absorbed in plasma itself has no influence on the phenomenon, since the condition of radiation $c'(\omega) < v$ is satisfied only for short enough plasma waves ($\lambda < D$), the damping coefficient of which is small in comparison with their frequency.

agreement is so good, rather than with explaining existing disagreements. »

Turning again to ordinary plasma I would like to emphasize, that the absorption of plasma waves in the plasma itself is conditioned by a reverse Vavilov-Čerenkov effect.

Ordinarily the necessary condition for a marked absorption of waves is the existence of a resonance between the frequency of the wave and a frequency of the absorbing system, e.g. an atom. Thus a free electron, which in distinction to a bound electron possesses no eigen-frequency, performs in the field of a wave periodic oscillations, alternatively acquiring and again losing kinetic energy and thus producing no substantial absorption.

But there exists also another non-resonant mechanism of absorption. If the velocity v of a free electron is greater than that of the wave (v > c'), then the projection of the velocity of the electron on the direction of propagation of the wave v cos Q may become equal to the velocity of the wave:

$$v \cos Q = c' \qquad (7)$$

In this case the electron so to say rides on the crest of the wave, being exposed to a force, the direction of which does not alter in time, and thus continually absorbs energy from the wave until its velocity increases so much, that it drops out of phase with the wave.

Such is the mechanism of absorption of plasma*; the condition (7), which sorts out those plasma electrons which take part in the process of absorption, is identical with the fundamental condition (2) for radiation**.

The damping coefficient y of plasma waves was first calculated by Landau” in 1946. Changing the notations used by Landau one can present the cxponential term in Landau's formula in the following form

$$\gamma \backsim \exp\left(-\frac{mu^2}{2\varkappa T}\right) \qquad (8)$$

* In principle this mechanism of absorption was indicated as long ago as 1949 by Bohm and Gross[11]. The work of these authors is intimately connected with earlier work of A. Vlasov. A detailed and a very lucid mathematical treatment of this subject was presented by R. Z. Sagdeev and V. D. Shafranov at the Geneva Atoms for Peace Conference last September.

** Radiation takes place if there is say one electron of velocity $\vec{v}$ or a cluster of such electrons, the dimensions of the cluster being small in comparison with the length of the wave radiated. If however electrons of a given velocity $\vec{v}$ are distributed continuously in space, then they do not radiate, since their wave-fields are destroyed by mutual interference. But they do absorb.

where $u = \omega_0/k$. In the range of validity of Landau's formula ω_0/k equals the velocity c' of the wave in question.

Therefore according to (8) the damping of a plasma wave is proportional to the density of plasma electrons, possessing according to Maxwell's law a velocity u, equal to the velocity of the wave. This is in exact correspondence to the mechanism of absorption just indicated.

In a recent paper on the mechanism of the sporadic solar radio-emission Ginzburg and Zhelesniakov[10] (1958) applied and extended the theory outlined above to a new and very interesting domain of physics, the foundations of which were laid in Sweden by Professor Alfvén. In particular they have shown that the known instability of a beam of charged particles traversing plasma, is from a quantum theoretical point of view due to the negative absorption of plasma waves by the beam of particles (the *induced* radiation of waves by the beam particles prevailing over the true absorption).

Before finishing I would like to mention one problem, which plays a rather important role in the present fascinating world-wide effort to harness thermonuclear reactions for peaceful uses - the problem how to heat the plasma. First stages of heating can be easily achieved by exciting an electric current in the plasma. However, the cross-section for Coulomb collisions of charged particles decreases inversely to the fourth power of their relative velocities and in a hot and rarefied plasma these collisions become so rare as to become negligible. Evidently heating by electric currents thus becomes impracticable: only a very small part of the energy of the ordered motion of plasma electrons, excited by an external field, is under these conditions converted into Joule heat.

Many different methods to achieve further heating of the plasma are now being discussed, e.g. the so-called magnetic pumping. I wish to make some remarks on only two such methods, intimately connected with our subject.

First, the heating by a beam of fast charged particles, injected into plasma from outside, is in principle feasible even if the plasma is hot and rarefied. Although in such a plasma energy losses of fast particles due to close collisions become negligible, coherent energy losses, described earlier, are independent of the collision cross-section and become all-important.

It is necessary to stress in this connection two points. First, the heating can in principle be achieved by a beam of fast charged particles travelling not in the plasma itself, but outside it and parallel to its surface. In fact, as we have seen, coherent energy losses are due to the emission of plasma waves by the fast particles. Now, those of these waves, the length of which is large in comparison with the distance of the beam from the surface of the plasma, will be excited by an external beam much to the same degree as by a beam traversing plasma. The possibilities offered by an external beam were first pointed out by L. Mandelstam for the case of the ordinary Vavilov-Čerenkov radiation. Later Ginzburg[12] (1947) proposed a method of generating micro-radiowaves by means of fast particles travelling along the surface of an appropriate dielectric or in a tunnel bored through the dielectric.

The second point is that if the beam consists of a succession of separate clusters of charged particles, then all the particles of each cluster will generate coherently those of the plasma waves, the length of which is large in comparison with the dimensions of the clusters. Therefore the intensity of these waves will be proportional not to the number of particles in a cluster, but to the square of this number. Evidently this offers the possibility of enhancing the radiation and the heating effect of a beam very considerably.

Let us now turn to another possible method of heating. Morozov[13] (1958) has recently calculated the excitation of so-called magneto-acoustic waves in a magnetic plasma (i.e. a plasma exposed to a constant external magnetic field) by an electric ring-current, moving with a sufficient velocity in a direction perpendicular to the plane of the ring-current. The current may move within the plasma - one can imagine a plasma ring, bearing a current, the ring being injected from outside into the plasma to be heated. Otherwise the current in question may be flowing outside of the plasma on the surface of the vessel containing it, such an external current being similar to an external beam of particles discussed above.

Generation of waves by a moving current is a special case of Vavilov-Cerenkov radiation. Morozov has shown that under certain conditions the absorption in plasma of magneto-acoustic waves produced in this way may in principle lead to a very considerable heating of the plasma. Of course the velocity of the current must exceed the velocity of the waves in question. One of the causes of high heating efficiency of a current is the coherence of the waves generated by its different elements. In this respect there exists an analogy between a current and a cluster of charged particles, the radiation of a current being proportional to the square of its strength.

There is another possible way of utilizing the Vavilov-Cerenkov radiation of a current. It is well known that currents excited in plasma, which in virtue of the pinch-effect are usually concentrated in a thin thread, are highly unstable. Therefore in practical applications it is often all-important to stabilize them. If the walls of the vessel containing plasma are conducting, then a displacement of the plasma current towards these walls will induce Foucault currents in them, and these currents will tend to repel the plasma current backwards. Methods of stabilization based on this phenomenon were independently proposed by physicists in different countries and were used in a number of thermonuclear experiments, but have proved to be not very satisfactory. Morozov and Soloviev[14] (1958) have recently proposed to construct the walls of vessels containing plasma not of conducting materials but of such materials, in which velocities of propagation of electromagnetic waves in an appropriate range of frequencies are as small as possible. If a current, flowing in plasma along the surface of such a wall, is displaced towards this surface with a velocity exceeding the velocity of propagation in the wall of waves of a certain frequency, then these waves will be radiated by the current into the wall. The recoil force acting on the current will tend to repel it from the wall and thus to stabilize the current.

I wish to emphasize that I have no definite opinion on possible advantages and disadvantages of methods of heating and of stabilization mentioned or on their technical feasibility. They were selected by me only as examples of possible applications of the general theory, which I have outlined in the beginning. The applications mentioned were necessarily confined to a very limited domain of physics.

I can only hope to have to some extent succeeded to convey to you the impression that there are further possibilities to apply this theory to new and interesting physical problems, and that work done on these lines may be useful in solving these problems or at least getting an insight into the general physical mechanism of some of the relevant phenomena.

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